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Coupled gap equations for the screening masses in the $SU(2)$ Higgs model

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Abstract. The complete set of static screening masses is determined for the $SU(2)$ Higgs model from one-loop coupled gap equations. Results from the version containing scalar fields both in the fundamental and adjoint representations are compared with the model arising when integration over the adjoint scalar field is performed. A non-perturbative and non-linear mapping between the couplings of the two models is proposed which exhibits perfect decoupling of the heavy adjoint scalar field. Also the alternative of a gauge invariant mass resummation is investigated in the high temperature phase.

1 Introduction

The finite temperature $SU(2)$ Higgs model was extensively studied in recent years in connection with the electroweak phase transition (EWPT) and baryon asymmetry generation in the standard model (see [1] for a review). Considerable progress was achieved in understanding the thermodynamics of the phase transition with the help of the method of dimensional reduction. In this approach the superheavy modes (i.e. the non-zero Matsubara modes with typical mass $\sim 2\pi T$) and the heavy A_0 field (with a mass $\sim gT$) are integrated out and the thermodynamics is described by an effective theory, the 3D $SU(2)$ Higgs model [2–4]. The properties of the phase transition and the screening masses were studied in great detail using lattice Monte Carlo simulations of the reduced model [5–9] and also by the Dyson–Schwinger (DS) technique in the full 4D theory [10,11] as well as in the effective 3D theory [12]. Lattice Monte Carlo simulations predict that the line of first order transitions ends for some Higgs mass $m_{\mathrm{H}} = m_{\mathrm{H}}^{\mathrm{c}}$ [6,8,9]. The same conclusion was obtained using the DS approach in [12] and the value of the critical mass $m_{\mathrm{H}}^{\mathrm{c}}$ was found to be close to the prediction of Monte Carlo simulations. Though the validity of one-loop gap equations was critically questioned [13,14], a recent two-loop calculation [15] has demonstrated that it is not an accident that the results of the one-loop level analysis are fairly close to the conclusions of the numerical simulations.

The possibility of dimensional reduction is based on the fact that in the full model there are different well separated mass scales $g^2T \ll gT \ll 2\pi T$ for small couplings g. Recent 4D Monte Carlo simulations of the finite temperature $SU(2)$ Higgs model [16,17] provide good non-perturbative tests for the validity of dimensional reduction. A detailed discussion of relating 4D and 3D results was published very recently in [18].

The purpose of the present paper is twofold. First, we would like to provide some non-perturbative evidence for the decoupling of the A_0 field from the gauge + Higgs dynamics in the vicinity of the phase transition. We are going to solve a coupled set of gap equations for the 3D fundamental + adjoint Higgs model. This model emerges when the non-static modes are integrated out in the full finite temperature Higgs system. Its predictions for the screening masses will be compared with those obtained by Buchmüller and Philipsen (BP) [12] in the 3D Higgs model (with only one scalar field in the fundamental representation) using the same technique. The main result of our investigation is a proposition for a non-perturbative and non-linear mapping between the two models ensuring quantitative agreement between the screening masses in a wide temperature range on both sides of the transition. This high quality evidence for the decoupling of the A_0 field at the actual finite mass ratios presumes, however, the knowledge of the "exact" value of the Debye screening mass, since for the proposed mapping its non-perturbatively determined value turns out to be essential.

Second, we wish to investigate the symmetric phase in more detail. There the Higgs and the Debye screening masses are both of the same order of magnitude $\sim gT$ and thus in that regime there is no a priori reason for the A_0 field to decouple. This circumstance makes the quantitative relation of the screening masses calculated in the 3D fundamental + adjoint Higgs model particularly interesting in the high-T phase. Here we are going to apply two different resummation techniques and check to what extent a non-perturbative mass hierarchy in this part of the spectra persists .

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All calculations of this paper are performed at one-loop accuracy, but the above mentioned signal [15] for the good numerical convergence of the masses determined in the DS scheme gives us confidence that the effects we find will appear also in improved treatments.

The presentation of our investigation proceeds as follows: in Sect. 2 we derive the coupled set of gap equations for the 3D fundamental + adjoint Higgs model and discuss some problems related to the formal decoupling of the adjoint Higgs field when its screening mass goes to infinity. In Sect. 3 we solve the coupled set of these equations numerically and estimate the variation in the screening masses and some critical parameters due to the presence of the adjoint Higgs field. In Sect. 4 we study the screening masses using an alternative gauge invariant resummation scheme, restricted in applicability to the symmetric phase. Finally, Sect. 5 presents our conclusions.

2 The extended gap equations

The Lagrangian of the three dimensional $SU(2)$ fundamental + adjoint Higgs model is [12,3]

$$\begin{aligned} L^{\mathrm{3D}} = & \mathrm{Tr}\Big[\frac{1}{2}F_{ij}\cdot F_{ij} + (D_i\Phi)^+(D_i\Phi) \\ & +\mu^2\Phi^+\Phi + 2\lambda(\Phi^+\Phi)^2\Big] \\ & +\frac{1}{2}(D_i\vec{A}_0)^2 + \frac{1}{2}\mu_{\mathrm{D}}^2\vec{A}_0^2 \\ & +\frac{\lambda_A}{4}(\vec{A}_0^2)^2 + 2c\vec{A}_0^2\mathrm{Tr}\Phi^+\Phi, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Phi &= \frac{1}{2}(\sigma 1 + \mathrm{i}\vec{\pi}\vec{\tau}), \quad D_i\Phi = (\partial_i - \mathrm{i}gW_i)\Phi, \\ W_i &= \frac{1}{2}\vec{\tau}\cdot\vec{W}_i. \end{aligned} \tag{2}$$

The relations between the parameters of the 3D theory and those of the 4D theory are perturbatively derived at the one-loop level [3]:

$$\begin{aligned} & g^2 = g_{\mathrm{4D}}^2 T, \quad \lambda = \left(\lambda_{\mathrm{4D}} + \frac{3}{128\pi^2}g_{\mathrm{4D}}^4\right)T, \\ & \lambda_A = \frac{17}{48\pi^2}g_{\mathrm{4D}}^4 T, \quad c = \frac{1}{8}g_{\mathrm{4D}}^2 T, \quad \mu_{\mathrm{D}}^2 = \frac{5}{6}g_{\mathrm{4D}}^2 T^2, \\ & \mu^2 = \left(\frac{3}{16}g_{\mathrm{4D}}^2 + \frac{1}{2}\lambda_{\mathrm{4D}}\right)T^2 - \frac{1}{2}\mu_{\mathrm{4D}}^2. \end{aligned} \tag{3}$$

If the integration over the A_0 adjoint Higgs field is performed we obtain the model investigated in [12] with parameters $\bar{g}, \bar{\lambda}, \bar{\mu}$. These couplings of the reduced theory are related to the parameters of the 3D fundamental + adjoint Higgs theory through the following relations:

$$\begin{aligned} & \bar{g}^2 = g^2\left(1 - \frac{g^2}{24\pi\mu_{\mathrm{D}}}\right), \quad \bar{\lambda} = \lambda - \frac{3c^2}{2\pi\mu_{\mathrm{D}}}, \\ & \bar{\mu}^2 = \mu^2 - \frac{3c\mu_{\mathrm{D}}}{2\pi}. \end{aligned} \tag{4}$$

In particular, we note that the $\bar{\mu}$ scale serves as the temperature scale of the fully reduced system, while μ is the scale for the system containing both the fundamental and the adjoint scalars. The two are related perturbatively by a constant shift.

In order to perform the actual calculations in the broken phase it is necessary to shift the Higgs field, $\sigma \to v+\sigma'$. After this shift and the gauge fixing (the gauge fixing parameter is denoted by ξ) the Lagrangian including the ghost terms assumes the form

$$\begin{aligned} L = & \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu\vec{W}_\mu)^2 + \frac{1}{2}m_0^2\vec{W}_\mu^2 \\ & +\frac{1}{2}(\partial_\mu\sigma')^2 + \frac{1}{2}M_0^2\sigma'^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \xi\frac{1}{2}m_0^2\vec{\pi}^2 \\ & +\frac{g^2}{4}v\sigma'\vec{W}_\mu^2 + \frac{g}{2}\vec{W}_\mu\cdot(\vec{\pi}\partial_\mu\sigma' - \sigma'\partial_\mu\vec{\pi}) \\ & +\frac{g}{2}(\vec{W}_\mu\times\vec{\pi})\cdot\partial_\mu\vec{\pi} + \frac{g^2}{8}\vec{W}_\mu^2(\sigma'^2+\vec{\pi}^2) \\ & +\lambda v\sigma'(\sigma'^2+\vec{\pi}^2) + \frac{\lambda}{4}(\sigma'^2+\vec{\pi}^2)^2 + \frac{1}{2}(D_i\vec{A}_0)^2 \\ & +\frac{1}{2}m_{\mathrm{D}0}^2\vec{A}_0^{\,2} + \frac{\lambda_A}{4}(\vec{A}_0^{\,2})^2 + 2cv\sigma'\vec{A}_0^{\,2} \\ & +c\vec{A}_0^{\,2}(\sigma'^2+\vec{\pi}^2) + \partial_\mu\vec{c^*}\cdot\partial_\mu\vec{c} + \xi m_0^2\vec{c^*}\cdot\vec{c} \\ & +g\partial_\mu\vec{c^*}\cdot(\vec{W}_\mu\times\vec{c}) + \xi\frac{g^2}{4}v\sigma'\vec{c^*}\cdot\vec{c} + \xi\frac{g^2}{4}v\vec{c^*}\cdot(\vec{\pi}\times\vec{c}) \\ & +\frac{1}{2}\mu^2v^2 + \frac{1}{4}\lambda v^4 + \frac{1}{2}(\mu^2+\lambda v^2)(\sigma'^2+\vec{\pi}^2) \\ & +v(\mu^2+\lambda v^2)\sigma', \end{aligned} \tag{5}$$

where the following notations were introduced for the tree-level masses: $m_0^2 = (1/4)g^2v^2$ (the vector boson mass), $M_0^2 = \mu^2 + 3\lambda v^2$ (the Higgs mass) and $m_{\mathrm{D}0}^2 = \mu_{\mathrm{D}}^2 + 2cv^2$ (the Debye mass). The last two terms of (5) arise from the Higgs potential after the shift in the Higgs field σ. For $\mu^2 < 0$, they vanish if one expands around the classical minimum $v^2 = -\mu^2/\lambda$. In general, however, these terms have to be kept [12].

In order to obtain the coupled gap equations one replaces the tree-level masses by the exact masses:

$$\begin{aligned} & m_0^2 \to m^2 + \delta m^2, \quad M_0^2 \to M^2 + \delta M^2, \\ & m_{\mathrm{D}0}^2 \to m_{\mathrm{D}}^2 + \delta m_{\mathrm{D}}^2, \end{aligned} \tag{6}$$

and treats the differences $\delta m^2 = m_0^2 - m^2, \delta M^2 = M_0^2 - M^2, \delta m_{\mathrm{D}}^2 = m_{\mathrm{D}0}^2 - m_{\mathrm{D}}^2$ as counterterms. The exact Goldstone and ghost masses are both equal to $\xi^{1/2}m$, where m is the exact gauge boson mass. The gauge invariance of the self-energies of the Higgs and gauge bosons is ensured by introducing appropriate vertex resummations. Their explicit formulae can be found in [12]. In the present extended model, a resummation of the Higgs–A_0 vertex would be also necessary if the gauge invariance of the A_0 self-energy is to be ensured. Then the only source of the gauge dependence which would remain is the equation for the vacuum expectation value v.

All these resummations are equivalent to working with the following gauge invariant Lagrangian:

$$L_I^{3\mathrm{D}} = \frac{1}{4}\vec{F}_{ij}\cdot\vec{F}_{ij} + \mathrm{Tr}\left((D_i\Phi)^+ D_i\Phi - \frac{1}{2}M^2\Phi^+\Phi\right) + \frac{1}{2}(m_\mathrm{D}^2 - \frac{8cm^2}{g^2})\vec{A}_0^2 + \frac{g^2M^2}{4m^2}\mathrm{Tr}(\Phi^+\Phi)^2 + 2c\vec{A}_0^2\mathrm{Tr}\Phi^+\Phi. \quad (7)$$

In this Lagrangian one shifts the Higgs field around its classical minimum $\sigma \to \sigma' + \frac{2m}{g}$ and adds the corresponding gauge fixing and ghost terms [12]. Shortly, we shall argue that the A_0–Higgs vertex resummation arising from the replacement of v by $2m/g$ when the scalar field is shifted in the last term of the above Lagrangian destroys the mass hiearchy between the heavy A_0 and the light gauge and Higgs fields. Therefore, in this paper we have to give up the full gauge independence of the resummation scheme. The numerical solution to be presented below shows that the gauge dependence of the $A_0 - -\Phi$ vertex in our resummation scheme introduces only a minor additional gauge dependence beyond that of the equation for the vacuum expectation value [12] appearing below in (14).

The coupled set of gap equations is constructed from that of [12] by adding the contributions due to the presence of the adjoint Higgs field. The self-energy contributions for the 3D fundamental Higgs and for the 3D adjoint Higgs model were already calculated in [12] and [19], respectively. Below we list only the additional contributions to the self-energies, which all contain at least one A_0–Φ vertex (the corresponding diagrams are listed in Appendix A). We emphasize once again that no resummation of the A_0–Φ vertex was applied.

The additional contribution to the self-energy of the A_0 field coming from Higgs, Goldstone, gauge and ghost fields (diagrams **a**–**i**) is

$$\delta\Pi_{A_0}^{H,G,gh}(p,m,M,m_\mathrm{D}) = -\frac{4cv^2(\mu^2+\lambda v^2)}{M^2} + \frac{3cgv}{\pi}\left(\frac{M}{4m}+\frac{m^2}{M^2}\right) - \frac{cM}{2\pi} + \frac{3\sqrt{\xi}}{4\pi}(gv-2m) + \frac{4c^2v^2}{\pi}\left[\frac{3}{2}\frac{m_\mathrm{D}}{M^2} - \frac{1}{p}\arctan\frac{p}{m_\mathrm{D}+M}\right]. \quad (8)$$

There is also an additional contribution to the gauge boson self-energy coming from the adjoint Higgs field (diagram m):

$$\delta\Pi_T^H(p,m,M,m_\mathrm{D}) = \frac{3cg}{2\pi}\frac{mv}{M^2}m_\mathrm{D}. \quad (9)$$

The contribution of $\vec{A}_0$ to the Higgs self-energy (diagrams **j**–**l**) is the following

$$\delta\Pi_H^{A_0}(p,m,m_\mathrm{D}) = -\frac{3m_\mathrm{D}c}{2\pi} - \frac{6c^2v^2}{\pi}\frac{1}{p}\arctan\frac{p}{2m_\mathrm{D}} + \frac{9gcv}{4\pi m}m_\mathrm{D}. \quad (10)$$

Making use also of the pieces of the self-energies calculated in [12,19] we write down a set of coupled on-shell gap equations for the screening masses of the magnetic gauge bosons, fundamental Higgs and adjoint A_0 fields in the form

$$m^2 = \Pi_T(p=\mathrm{i}m,m,M) + \delta\Pi_T^{A_0}(p=\mathrm{i}m,m_\mathrm{D}) + \delta\Pi_T^H(p=\mathrm{i}m,m,M,m_\mathrm{D}), \quad (11)$$

$$M^2 = \Sigma(p=\mathrm{i}M,m,M) + \delta\Pi_H^{A_0}(p=\mathrm{i}M,m,m_\mathrm{D}), \quad (12)$$

$$m_\mathrm{D}^2 = \Pi_{00}(p=\mathrm{i}m_\mathrm{D},m,m_\mathrm{D}) + \delta\Pi_{A_0}^{H,G,gh}(p=\mathrm{i}m_\mathrm{D},m,M,m_\mathrm{D}), \quad (13)$$

where Π_T and Σ are defined by (17) and (18) of [12]. $\delta\Pi_T^{A_0}$ and Π_{00} were presented in (7) and (8) of [19].

If on the right hand side of the third equation one inserts the tree-level masses, the next-to-leading order result of [20] is recovered for the Debye mass in the $SU(2)$ Higgs model.

The equation for the vacuum expectation value makes the set of the above three equations complete:

$$v(\mu^2+\lambda v^2) = \frac{3}{16\pi}g\left(4m^2+\sqrt{\xi}M^2+\frac{M^3}{m}\right) + \frac{3c}{2\pi}vm_\mathrm{D}. \quad (14)$$

It is important to notice that this equation can be rewritten as

$$v(\mu_\mathrm{eff}^2+\lambda v^2) = \frac{3}{16\pi}g\left(4m^2+\sqrt{\xi}M^2+\frac{M^3}{m}\right), \quad (15)$$

with

$$\mu_\mathrm{eff}^2 = \mu^2 - \frac{3c}{2\pi}m_\mathrm{D}. \quad (16)$$

This equation is formally identical to the equation of BP for the vacuum expectation value [12]. On the basis of this observation, we expect that the main effect of the A_0 integration is the above shift in the μ^2 scale. Since m_D is itself a non-trivial function of μ this non-perturbative mapping is also nonlinear.

A very similar set of equations could be derived for the case of the gauge invariant resummation of the A_0–Φ vertex. They are listed in Appendix B. For instance, one would write in the last term on the right hand side of (14) $2m/g$ in place of v, which would suggest a different redefinition of the temperature (μ^2) scale:

$$\tilde{\mu}_\mathrm{eff}^2 = \mu^2 - \frac{3c}{\pi g}\frac{mm_\mathrm{D}}{v}. \quad (17)$$

If the tree-level masses are inserted into this redefinition it gives the usual relation between the mass parameters of the full static and the A_0 reduced models (see (4)).

3 Numerical results

The main goal of the present investigation is to propose a scheme of solution for the full static Higgs model (1) which

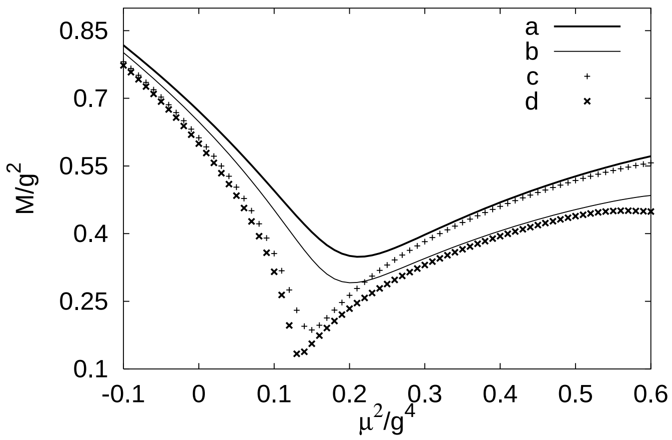

Fig. 1. The Higgs mass in units of g^2 as a function of μ^2/g^4 calculated at $\lambda/g^2 = 1/8$ using the gauge invariant A_0–Φ vertex resummation version of the gap equations. Shown are the Higgs mass derived in the full static theory in the Landau **a** and in the Feynman gauge **b** and the Higgs mass values in the A_0 reduced theory in the Landau gauge **c** and in the Feynman gauge **d**. The μ^2 shift indicated by (4) was applied

reproduces the BP solution of the reduced static model (with A_0 integrated out). The existence of such a solution is made plausible by the Appelquist–Carazzone theorem [21], but by no means it is trivial to construct it for two obvious reasons. The decoupling theorem is valid only for infinitely different mass scales, while the m_D/m, m_D/M ratios are finite in the realistic case. There are corrections to the theorem even if we would be able to compare the exact values of the corresponding masses calculated in the two models for the perturbatively related values of the couplings. The second source of deviations comes from the resummation applied in the process of the perturbative solutions. It is not clear which resummed solution of the full static model would correspond to the BP resummed approximate solution of the reduced 3D effective model at one-loop level.

Though the construction of a good quality correspondence is a very difficult task, it is a necessary effort if one wishes to go beyond the "existence proof" of the decoupling in case of the resummed solutions.

We have to admit that it would be much easier to assess the status of A_0 decoupling and the quality of the BP solution if the exact (Monte Carlo) solution of the model (1) would be available. However, Monte Carlo simulations of the gauge + fundamental + adjoint Higgs system are extremely difficult to realize (see discussion in [5]). Therefore, our present construction can be considered a first detailed attempt to establish quantitative arguments for the A_0 decoupling.

Our first attempt at solving the full static model followed the gauge invariant vertex resummation procedure employed also by the BP solution of the A_0 reduced model. In Fig. 1 the results of the two solutions for the Higgs mass M are displayed taking into account the perturbative mapping (4) between the parameters of the two models. The deviations are large, especially in the critical region. We arrived at a negative conclusion: The gauge invariantly resummed one-loop solutions of the gap equations of the two models do not correspond to each other if the perturbative A_0 integration is correct.

We have also tried to compare the predictions of the full static and the reduced models in the case when the mass parameter of the reduced model is chosen according to (17). Such a non-perturbative mapping between the parameters of the two models somewhat improves the situation deep in the broken phase; however, near the crossover region the values of the masses calculated in the two models differ considerably. We conclude that if a gauge invariant resummation of the A_0–Φ vertex is used we are not able to find a physically motivated relation between the parameters of the full static and the reduced models with the help of which the two models give acceptably close mass predictions. Therefore, we will no further discuss the fully gauge invariant resummation scheme but turn to the discussion of the results obtained in the case when the A_0–Φ vertex is left unresummed.

If the A_0–Φ vertex is left unresummed, a very simple expectation emerges concerning the effect of the A_0 integration on the mass spectra, as was disscused on the basis of (16) in the previous section. Therefore, we will first compare the predictions for the Higgs and gauge boson masses from the coupled gap equations (11)–(14) of the 3D fundamental + adjoint Higgs model with those obtained in the A_0 reduced theory, the 3D Higgs model [12]. The corresponding Higgs masses are shown in Fig. 2 using two different gauges. The results obtained in the A_0 reduced theory are displayed after the shift required by (4) is performed. As one can see the difference between the full and the reduced theory is still visible in the vicinity of the crossover. In this region the relative difference between the predictions of the full and the reduced theory is about 20%.

Our proposal to resolve this relatively large deviation is to introduce a more complicated relationship between the couplings. Having gained intuition from (16), we have plotted the mass predictions for the Higgs field derived from our full set of equations against the results of BP calculated for couplings taken from (4) with the replacement $\mu_\mathrm{D} \to m_\mathrm{D}$:

$$g^2_\mathrm{eff} = g^2(1 - \frac{g^2}{24\pi m_\mathrm{D}}), \quad \lambda_\mathrm{eff} = \lambda - \frac{3c^2}{2\pi m_\mathrm{D}},$$
$$\mu^2_\mathrm{eff} = \mu^2 - \frac{3c}{2\pi} m_\mathrm{D}. \tag{18}$$

The non-trivial nature of this replacement becomes clear from Fig. 3 where the μ^2 dependence of m_D is displayed. Clearly, its non-trivial μ^2 dependence is most expressed in the neighborhood of the phase transformation (crossover) point $\mu^2/g^4 \in (0.1–0.2)$. The application of this mapping to the data obtained from the model containing both the fundamental and the adjoint representation leads to a perfect agreement of the two data sets for large values of λ/g^2. For smaller values of λ/g^2 $(1/32, 1/64)$ the mapping (18)

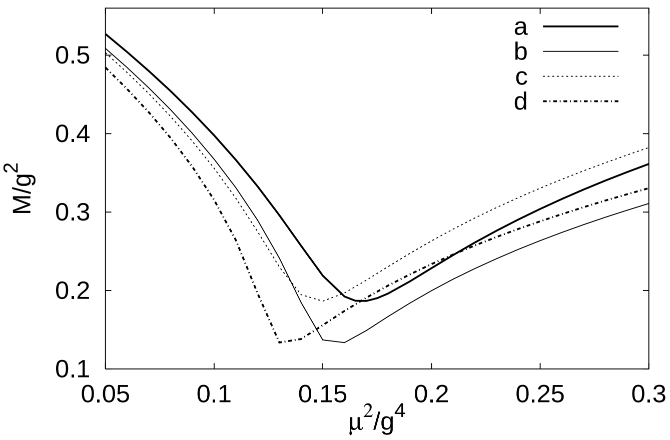

Fig. 2. The Higgs boson masses at $\lambda/g^2 = 1/8$ (crossover region) in units of g^2 as a function of μ^2/g^4 in the 3D fundamental + adjoint Higgs model and in the 3D $SU(2)$ Higgs (A_0 reduced) theory. Shown are the Higgs mass in the full static theory in the $\xi = 0$ (Landau) gauge **a** and in the $\xi = 1$ (Feynman) gauge **b**, and the Higgs boson mass in the A_0 reduced theory in the $\xi = 0$ gauge **c** and in the $\xi = 1$ gauge **d**

works very well in the symmetric phase, but in the broken phase (4) seems to be the better choice.

We suspect that the tree-level piece in $m_{\rm D}$ arising from the Higgs effect should not be included into the correction of (4), since it is itself a tree-level effect. Therefore, we propose the following replacement in (18):

$$m_{\rm D} \to \sqrt{m_{\rm D}^2 - 2cv^2}. \quad (19)$$

In Fig. 4 it is obvious that a very good agreement could be obtained with this mapping between the Higgs mass predictions of the one-loop gap equations of the full static and the A_0 reduced theory for $\lambda/g^2 = 1/32$. The quality of the agreement on both sides of the phase transition is good, signalling that the influence of the "mini-Higgs" effect in the symmetric phase is negligible. Therefore, it is not surprising that for $\lambda/g^2 = 1/8$ the same quality of agreement is obtained as before.

It is important to notice that there is a strong gauge parameter dependence in the symmetric phase and in the vicinity of the crossover. The variations due to the change in the gauge are equal in the full and in the reduced theory, which indicates that the additional gauge dependence, introduced by the gauge non-invariant resummation of the A_0 field is negligible. The mapping (19) performs equally well in the Landau and in the Feynman gauge.

Other quantities which are worth of considering for the comparison of the full 3D and the reduced theories are λ_c/g^2, the endpoint of the first order transition line and μ_+/g^2, the mass parameter above which the broken phase is no longer metastable. The values of μ_+^2/g^4 for different scalar couplings and different gauges in the full and in the reduced theory are summarized in Table 1. Here the mapping (18) could be implemented only by extrapolating from smaller μ^2/g^4, since the endpoints of metastability do not correspond to each other, and in some cases $m_{\rm D}$ could not be determined from the gap equations. Also here for larger values of λ/g^2 the application of (18) led to an improved agreement between the endpoint μ_+^2/g^4 values, while for $\lambda/g^2 = 1/48, 1/64$ the mapping (4) works better. In the table we have displayed μ_+^2/g^4 values of the A_0 reduced theory shifted perturbatively and with help of the best performing non-perturbative mapping (19). For both gauges the latter agrees with the μ_+^2/g^4 values of the full static theory very well.

Table 1. Values of μ_+^2/g^4 in the full static theory (A), in the perturbatively reduced theory (B) and in the reduced theory obtained using non-perturbative matching described in the text (C). Calculations were done in the Landau ($\xi = 0$) and in the Feynman ($\xi = 1$) gauges

λ/g^2	A		B		C	
	$\xi = 0$	$\xi = 1$	$\xi = 0$	$\xi = 1$	$\xi = 0$	$\xi = 1$
1/32	0.1516	0.1423	0.1426	0.1341	0.1499	0.1405
1/48	0.1647	0.1558	0.1627	0.1541	0.1637	0.1546
1/64	0.1841	0.1750	0.1881	0.1808	0.1875	0.1792

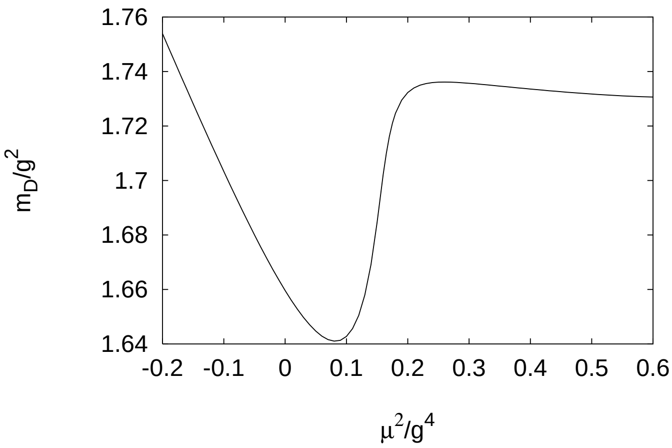

Fig. 3. The μ^2 dependence of the Debye mass for $\frac{\lambda}{g^2} = 1/8$

The endpoint of the 1st order line in the Landau gauge in the 3D Higgs theory was found at $\lambda_c/g^2 = 0.058$. The corresponding critical scalar coupling in the full 3D theory is within the 1% range. In Feynman gauge we find $\lambda_c/g^2 = 0.078$ for the A_0 reduced theory and the corresponding value for the full 3D theory lies again very close to it. Thus, the A_0 field has almost no effect on the position of the endpoint. The strong gauge dependence of λ_c indicates, however, that higher order corrections to this quantity are important.

The depth of the gauge dependence of the screening masses is pronounced, even more so in the symmetric phase ($\mu^2/g^4 > 0.3$). For example the value of the gauge boson mass is roughly $0.28g^2$ in the symmetric phase for the Landau gauge. The corresponding value in the Feynman gauge is about $0.22g^2$. The gauge dependence of the gauge boson mass is somewhat weaker at the two-loop

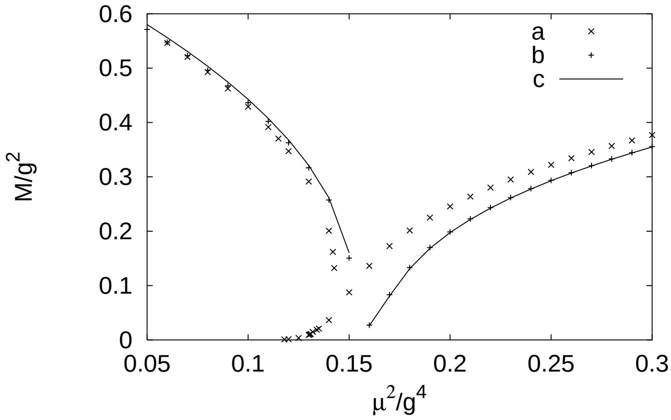

Fig. 4. The Higgs boson mass in units of g^2 as a function of μ^2/g^4 calculated at $\lambda/g^2 = 1/32$ in the Landau gauge in the full static theory and in the A_0 reduced theory. Shown are the Higgs mass in the reduced theory obtained by perturbative reduction **a**, in the reduced theory obtained by non-perturbative matching (cf. (18) and (19)) **b** and in the full static theory **c**

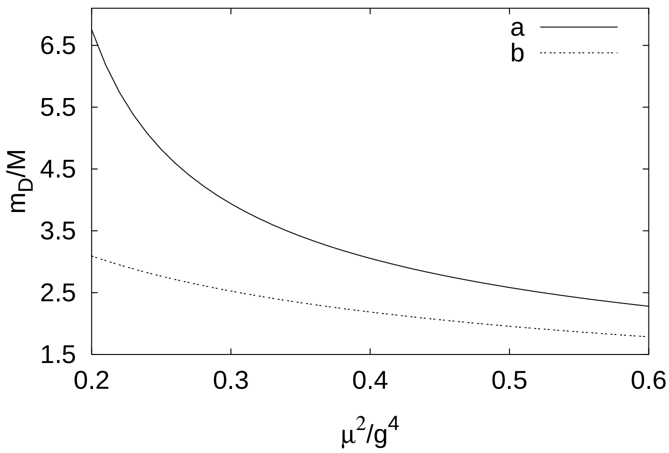

Fig. 5. The ratio of the Debye and the fundamental Higgs masses for $\frac{\lambda}{g^2} = 1/8$ calculated from gap (21) **a** and the leading order result **b**

level [15]. It should be also noticed that the gauge boson mass depends weakly on the parameters of the scalar sector (μ, $\mu_{\rm D}$, λ, λ_A). This fact was also noticed in previous investigations [12,19].

4 Screening masses in the symmetric phase with a gauge invariant resummation scheme

The main motivation for the present investigation was to gain insight into the decoupling of the dynamics of the fundamental and the adjoint Higgs fields. The degree of the decoupling is expected to depend on the mass ratio of the fundamental and adjoint Higgs fields. In the symmetric phase both masses are of the same order in magnitude (e.g. $\sim gT$). Therefore, the hierarchy of the A_0 and Higgs masses can only be present due to numerical prefactors. The persistence of the perturbatively calculated ratio should be checked in any non-perturbative approach.

As we have seen in the previous section the gauge dependence in the symmetric phase is too strong in the applied schemes to give a stable estimate for the mass ratio of the fundamental and the adjoint Higgs fields. A reliable non-perturbative estimate for the Higgs mass deep in the symmetric phase (defined through the pole of the propagator) is even more interesting because it was not measured so far on lattice. Therefore, in this section we will investigate a coupled set of gap equations in the symmetric phase which is based on the gauge invariant resummation scheme of Alexanian and Nair (AN) [22]. In this approach one can avoid any vacuum expectation value for the Higgs field in the symmetric phase and because of this fact this approach is gauge invariant.

In order to derive the one-loop gap equations for the Higgs model in the AN scheme one has to add the following terms to the original Lagrangian:

$$\delta L = \frac{1}{2} m^2 W_i (\delta_{ij} + \frac{\partial_i \partial_j}{\partial^2}) W_j + f^{abc} V_{ijk} W_i^a W_j^b W_k^c - \frac{1}{2\xi} \partial_i W_i (1 - m^2 \frac{1}{\partial^2}) \partial_j W_j. \quad (20)$$

The first term in this expression is the mass term, the second corresponds to a specific vertex resummation, where the explicit expression for V_{ijk} could be find in [22]. Finally, the last term is the gauge fixing term. For the coupled gap equations one has to re-evaluate those self-energy diagrams of the gauge, Higgs and A_0 fields which involve the modified gauge propagators from (20). Straightforward calculations lead to the following equations:

$$m^2 = Cg^2 m + \frac{g^2 m}{4\pi}\Big(2f(m_{\rm D}/m) + f(M/m)\Big), \quad (21)$$
$$M^2 = \mu^2 + \frac{1}{4\pi}\left(\frac{3}{4} g^2 M F(M/m) - 6\lambda M - 6cm_{\rm D}\right),$$
$$m_{\rm D}^2 = \mu_{\rm D}^2 + \frac{1}{4\pi}\Big(2g^2 m_{\rm D} F(m_{\rm D}/m) - 5\lambda_A m_{\rm D} - 8cM\Big),$$

where $C = (1/(4\pi))(21/4 \ln 3 - 1)$ [22] and the following function was introduced:

$$f(z) = -\frac{1}{2} z + \left(z^2 - \frac{1}{4}\right) \operatorname{arctanh} \frac{1}{2z}, \quad (22)$$
$$F(z) = -1 - \frac{1}{z} + \left(4z - \frac{1}{z}\right) \ln(1 + 2z). \quad (23)$$

Let us first discuss the ratio of the A_0 and the fundamental Higgs masses. In Fig. 5 this ratio is shown as calculated from (21) and is compared with the corresponding perturbative value. The μ interval in this plot corresponds to the temperature range relavant for the electroweak theory $T < 1\,\text{TeV}$. We have also analyzed the μ dependence of the fundamental Higgs mass alone in the full static and in

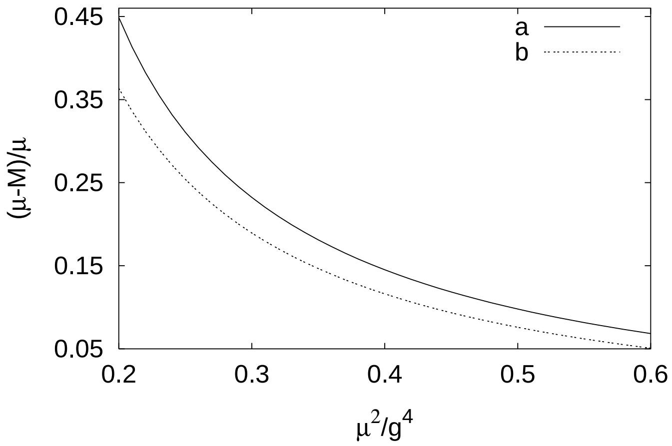

Fig. 6. The non-perturbative correction to the Higgs mass as a function of μ calculated from the full static **a** and from the A_0 reduced theory **b**

the A_0 reduced model. For μ^2/g^4 in the interval $(0.2-0.3)$ the result of the gauge invariant approach agrees fairly well with the masses obtained in the BP scheme. In Fig. 6 the difference between the Higgs masses calculated from the coupled set of gap equations (21) and the leading order perturbative result (M_0) is shown. As one can see, the non-perturbative correction to the Higgs mass is the largest for small μ and is decreasing as μ increases reaching the percent level for large enough μ.

The relative difference between the full static and the A_0 reduced theory, however, is slowly increasing as μ increases and the hierarchy between the A_0 and the Higgs masses becomes less pronounced as μ gets larger (see Fig. 6). The relative difference between the Higgs masses calculated in the full static and in the A_0 reduced theory varies between 20% for $\mu^2/g^4 = 0.2$ and 35% for $\mu^2/g^4 = 0.6$.

It is also important to notice that the A_0 field is not sensitive to the dynamics of the Higgs field. In particular it turns out that $m_{\rm D}$ depends weakly on μ and λ in the symmetric phase and its value is close to the corresponding value calculated in the 3D adjoint Higgs model. Let us notice that the magnetic mass in this resummation scheme also seems to be insensitive to the dynamics of the scalars; therefore, the magnetic and electric screening masses are close to their values determined in the pure $SU(2)$ gauge model [19].

5 Conclusions

The Appelquist–Carazzone (AC) theorem provides an important asymptotic basis for the derivation of reduced effective models, when fields with largely different masses appear in a field theoretical model. It states that in the infinite mass limit the n point functions of the light degrees of freedom can be calculated from an effective theory, in which the effect of the heavy fields is present only in the couplings. In the electroweak theory these effective models were determined perturbatively. In resummed perturbation theory for finite orders the fulfillment of the theorem cannot be checked on a diagram-by-diagram basis.

The comparative investigation of the screening masses of the full static and the A_0 reduced theories of the finite temperature $SU(2)$ Higgs model gives us a very valuable opportunity to study how well the AC theorem works under realistic mass ratios. In particular, in the symmetric phase of the theory we have seen that a non-perturbative coupling relation (18) is necessary to map almost perfectly the masses determined in the A_0 reduced model onto those found from the gap equations of the complete static effective model. The λ/g^2 range (1/64–1/8) has covered the regime of strong first order transitions to values where only a smooth crossover takes place. The correspondence between specific solution schemes, which is compatible with the AC theorem represents constructive evidence for the validity of the theorem.

The quality of the mapping did not depend on the gauge choice, which, however, strongly influences the actual values of the screening masses. Therefore, we have also applied a gauge invariant resummation scheme in the symmetric phase. The results show a larger $m_{\rm D}/M$ ratio than perturbatively predicted, which makes the basis for the A_0 reduction more solid.

In the broken symmetry phase the non-perturbative mapping as given by (18) does not work. The attempt to separate the non-perturbative change of the Debye mass from the result of the symmetry breaking led us to propose the mapping (19). It gave very satisfactory results for both the Higgs mass and the upper metastability edge μ_+/g^2 in the Higgs mass range $\lambda/g^2 \in (1/32, 1/8)$, when resummed one-loop solutions of different relevant models in specific schemes are calculated. We believe that our phenomenological observation opens the door towards a more refined physical understanding of the relationship of the couplings in the two models. This is necessary for the consolidation of the status of a non-perturbative A_0 decoupling from the static sector of the finite temperature Higgs theory.

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Appendix A

In Fig. 7 we list graphycally the additional diagrams contributing to the A_0 (**a**–**i**), the Higgs boson (**j**–**l**) and the vector boson **m** self-energies and the vacuum expectation value **n**.

Appendix B

The gap equations for the masses in the gauge invariant resummation scheme read

$$m^2 = m_0^2 + mg^2 f_B(m/M)$$

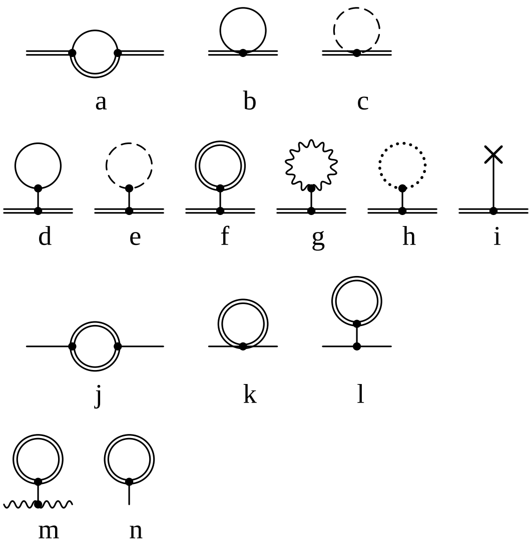

Fig. 7a–n.

$$+\frac{g^2}{2\pi}\left(-\frac{m_{\rm D}}{2}+\frac{4m_{\rm D}^2-m^2}{4m}\mathrm{arcth}\frac{m}{2m_{\rm D}}\right), \quad (24)$$

$$M^2 = M_0^2 + g^2 M F_B(m/M) - \frac{3}{2\pi}\left(\frac{4mc}{g}\right)^2\frac{1}{M}\mathrm{arcth}\frac{M}{2m_{\rm D}} - \frac{3}{2\pi}cm_{\rm D}, \quad (25)$$

$$m_{\rm D}^2 = m_{\rm D0}^2 + \frac{g^2}{\pi}\left[-\frac{m_{\rm D}}{2}-\frac{m}{2} + \left(m_{\rm D}-\frac{m^2}{4m_{\rm D}}\right)\ln\frac{2m_{\rm D}+m}{m}\right] - 8v(\mu^2+\lambda v^2)\frac{mc}{gM^2} + \frac{1}{\pi}cM\left(1+6\frac{m^3}{M^3}\right) + \frac{1}{\pi}\left(\frac{4mc}{g}\right)^2\left[\frac{3}{2}\frac{m_{\rm D}}{M^2}-\frac{1}{2m_{\rm D}}\ln\frac{2m_{\rm D}+M}{M}\right], \quad (26)$$

$$v(\mu^2+\lambda v^2) = -M^2\delta f_B(m/M) + \frac{3}{\pi}\frac{c}{g}mm_{\rm D}, \quad (27)$$

where the $\delta f_B(z) = f_B(z) - \bar{f}_B(z)$ and $\bar{f}_B(z), f_B(z), F_B(z)$ are defined by the (24), (30) and (31). of [12]:

$$\bar{f}_B(z) = \frac{1}{\pi}\left[\frac{63}{64}\ln 3 - \frac{1}{8} + \frac{1}{32z^3} - \frac{1}{32z^2} + \frac{1}{8z} + \frac{3}{4}z^2 - \left(\frac{1}{64z^4}-\frac{1}{16z^2}+\frac{1}{8}\right)\ln(1+2z)\right], \quad (28)$$

$$f_B(z) = \frac{1}{\pi}\left[\frac{63}{64}\ln 3 - \frac{1}{8} + \frac{1}{32z^3} - \frac{1}{32z^2} - \frac{1}{16z} - \frac{3\sqrt{\xi}}{16} - \left(\frac{1}{64z^4}-\frac{1}{16z^2}+\frac{1}{8}\right)\ln(1+2z)\right], \quad (29)$$

$$F_B(z) = \frac{1}{\pi}\left[-\left(\frac{3}{32}+\frac{9}{64}\ln 3\right)\frac{1}{z^2} + \frac{3}{16}\left(1-\frac{3}{2}\sqrt{\xi}\right)\frac{1}{z} - \frac{3}{8}z - \left(\frac{3}{8}z^2-\frac{3}{16}+\frac{3}{64z^2}\right)\ln\frac{2z+1}{2z-1}\right]. \quad (30)$$

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